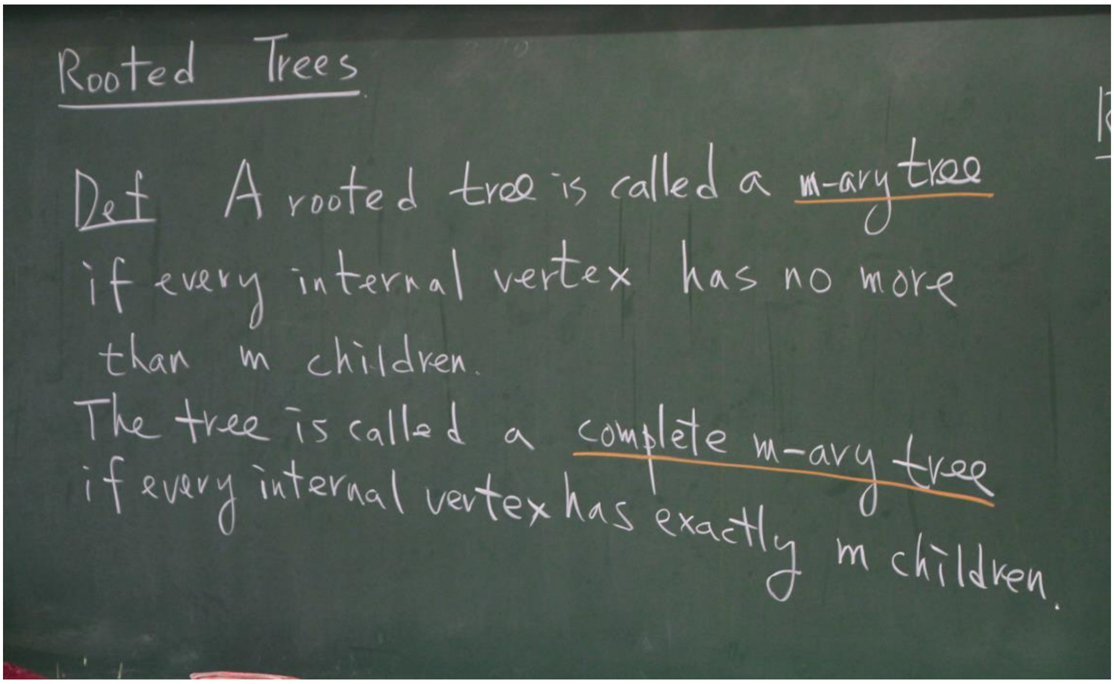
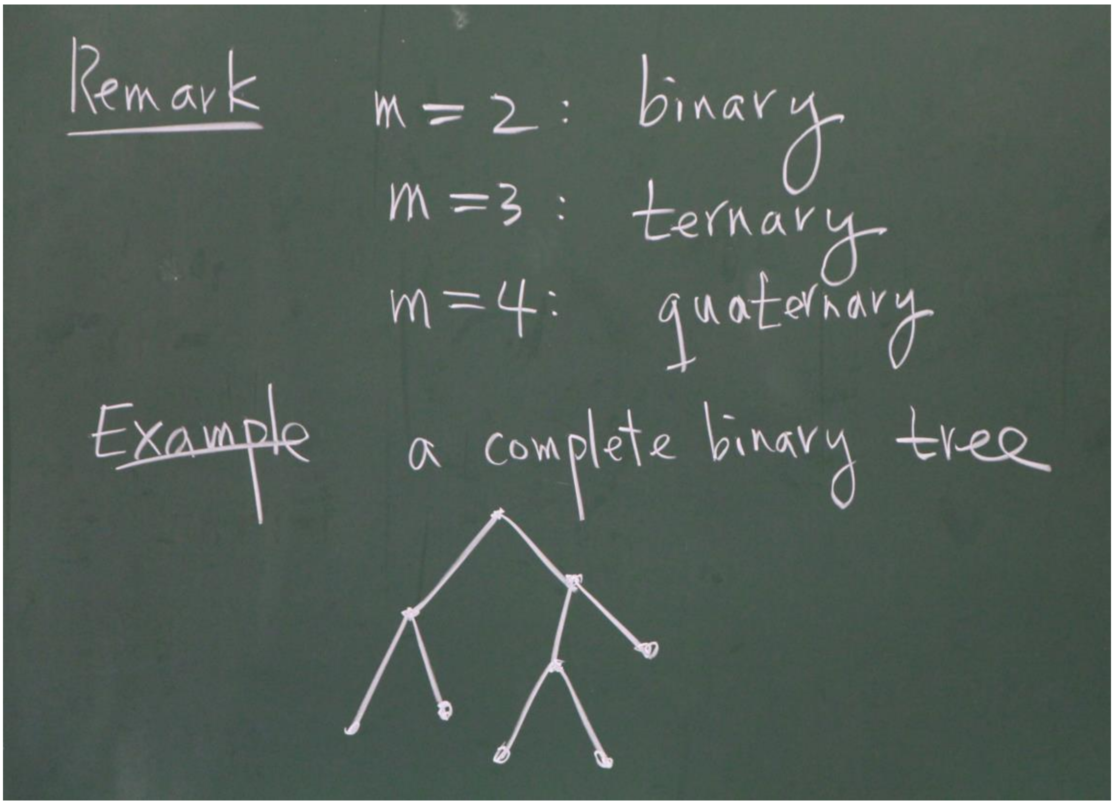
【10920 趙啟超教授離散數學 / 第 24 堂版書】





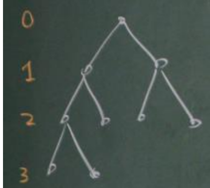
Property Let T be a complete m-ary tree with n vertices, in which there are l leaves and i internal vertices. Then

1. $n = mi + 1$.
2. $l = (m-1)i + 1$.
3. $i = \frac{l-1}{m-1} = \frac{n-1}{m}$.

Proof 1. Every vertex, except the root, is a child of an internal vertex. Since each of i internal vertices has m children, there are mi vertices in the tree other than the root. Therefore, the tree contains $n = mi + 1$ vertices.

2. $l = n - i = mi + 1 - i = (m-1)i + 1$.

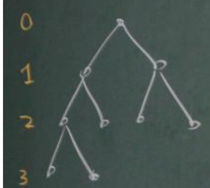
3. From 1 and 2, $i = \frac{l-1}{m-1} = \frac{n-1}{m}$. ∎



Def A rooted tree of height h is called **balanced** if the level number of every leaf is $h-1$ or h.

Property For an m-ary tree of height h with ℓ leves, $\ell \leq m^h$ and $h \geq \lceil \log_m \ell \rceil$.

Remark $\lceil x \rceil$ is the smallest integer $\geq x$.
$\lfloor x \rfloor$ is the largest integer $\leq x$.



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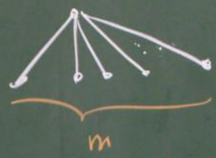
Proof The proof is by induction on h.
For $h = 1$, $\ell \le m = m^h$.



Now assume that the result is tree for all m-ary trees of height $< h$.
Consider the subtrees at vertices of level 1.
There are at most m such subtrees.

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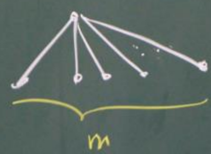

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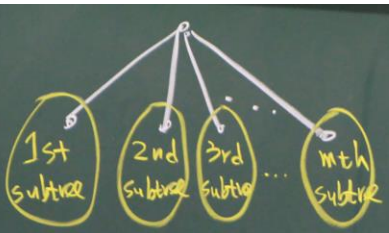
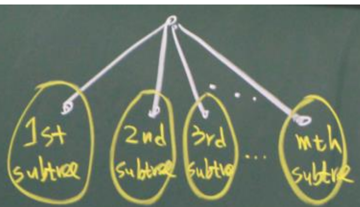


Each subtree has height $\leq h-1$.
So by the induction hypothesis, each subtree has $\leq m^{h-1}$ leaves.
Hence there are at most $m \cdot m^{h-1} = m^h$ leaves in T. This completes the induction argument.
Since h is an integer and $l \leq m^h$, $h \geq \lceil \log_m l \rceil$. $\blacksquare$



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Property For a balanced complete m-ary tree with ℓ leaves, $h = \lceil \log_m \ell \rceil$.

Proof Since each leaf is of level h or $h-1$ and the height is h, there is at least one leaf of level h. It follows that there are more than m^{h-1} leaves.
We then have $m^{h-1} < \ell \leq m^h$.

$$\Rightarrow \quad h-1 < \log_m \ell \leq h.$$

Hence $h = \lceil \log_m \ell \rceil$.

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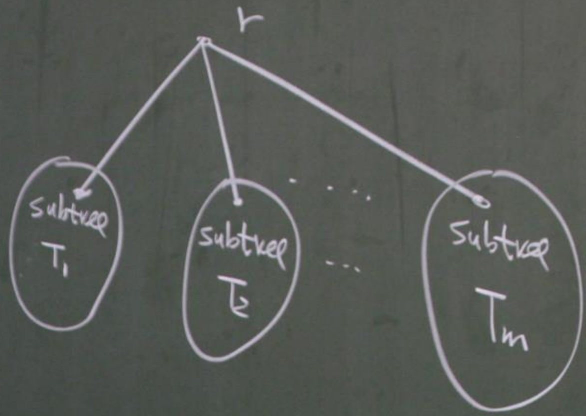
Tree Traversal

Consider a rooted tree T with root r.



Preorder traversal

1. Visit r
2. Traverse T_1 in preorder.
3. Traverse T_2 in preorder.

$\vdots$

$m+1$. Traverse T_m in preorder.

Preorder traversal

1. Visit r
2. Traverse T_1 in preorder.
3. Traverse T_2 in preorder.

⋮

$m+1$. Traverse T_m in preorder.

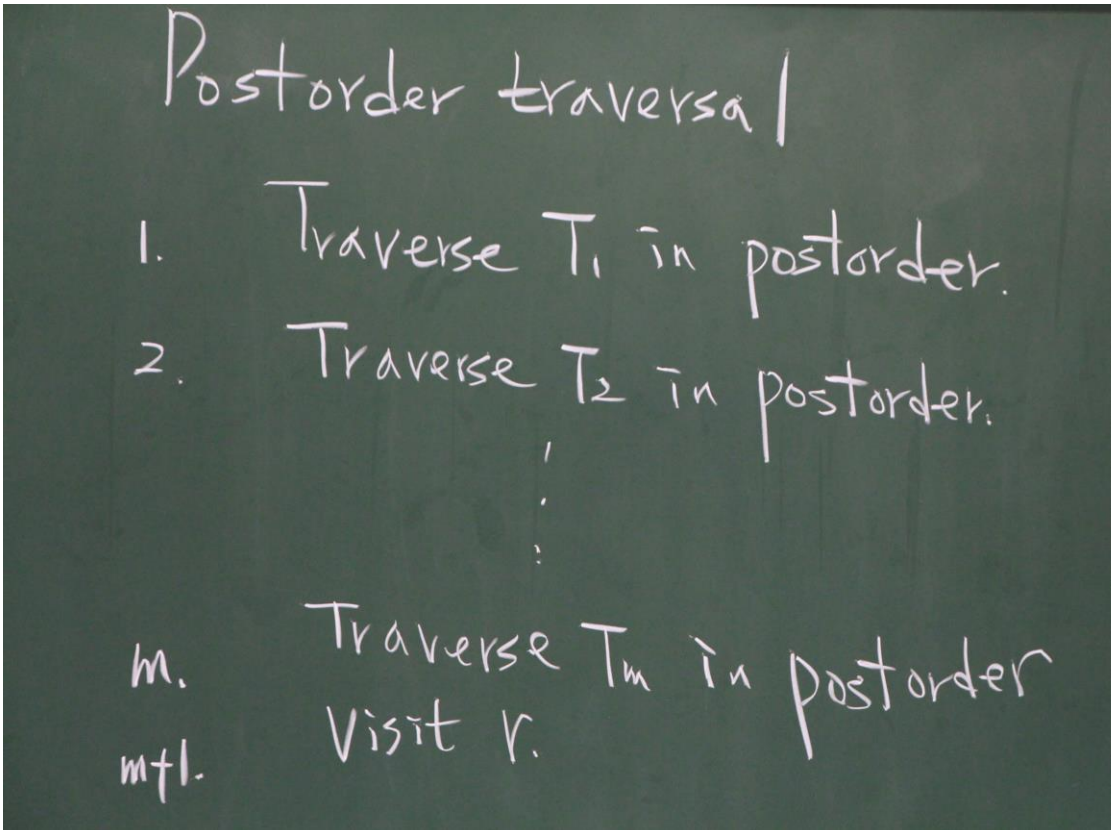
Postorder traversal

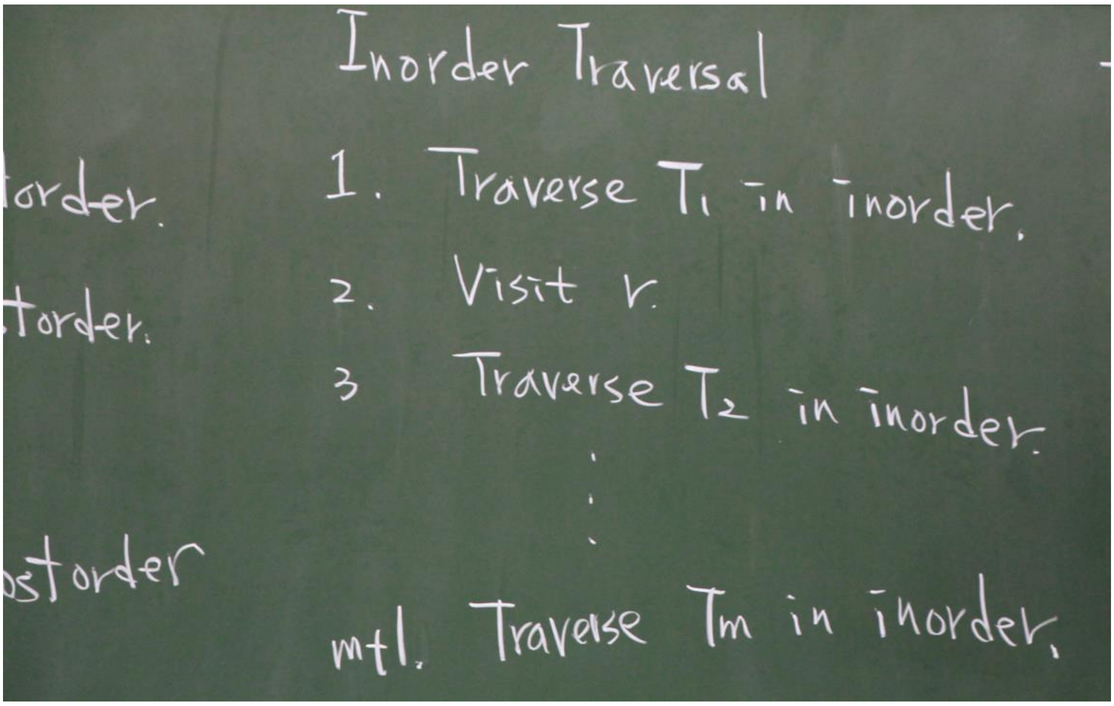
1. Traverse T_1 in postorder.
2. Traverse T_2 in postorder.

⋮

m. Traverse T_m in postorder

$m+1$. Visit r.


Postorder traversal
1. Traverse T_1 in postorder.
2. Traverse T_2 in postorder.
⋮
m. Traverse T_m in postorder
m+1. Visit r.


Inorder Traversal
1. Traverse T_1 in inorder.
2. Visit r.
3 Traverse T_2 in inorder.
⋮
m+1. Traverse T_m in inorder.

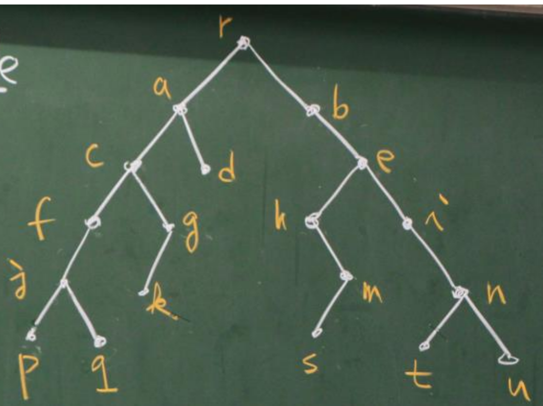
Inorder Traversal

1. Traverse T_1 in inorder.
2. Visit r.
3. Traverse T_2 in inorder.

⋮

$m+1$. Traverse T_m in inorder.

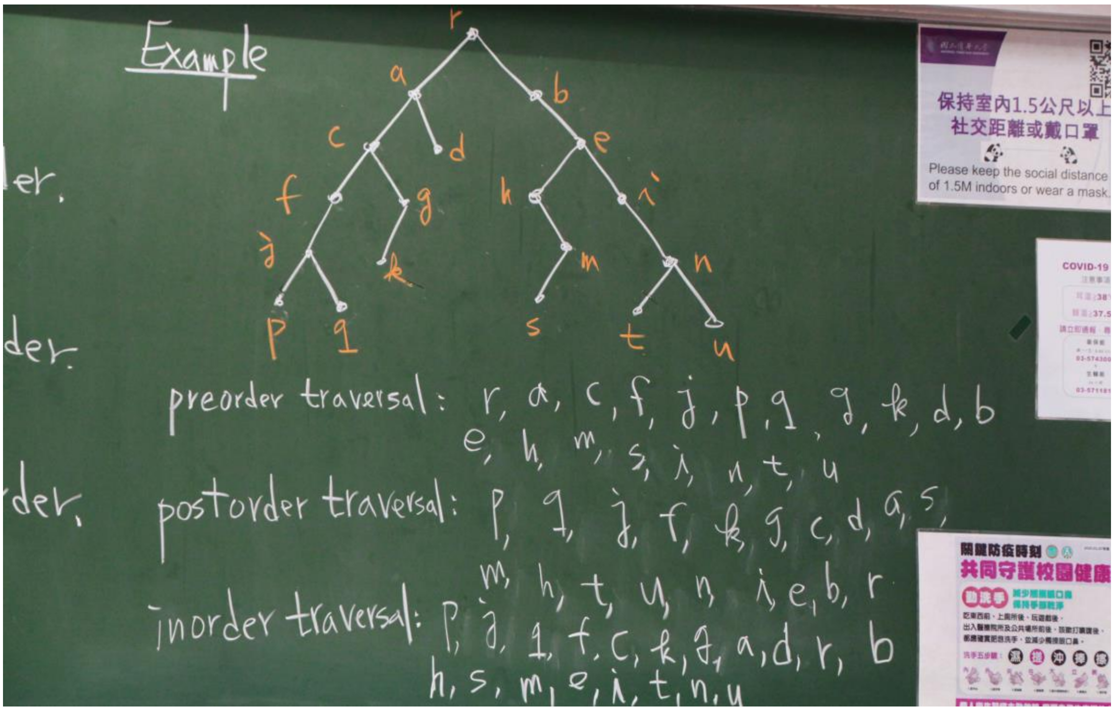
Example



preorder traversal: r, a, c, f, j, p, q, g, k, d, b, e, h, m, s, i, n, t, u

postorder traversal: p, q, j, f, k, g, c, d, a, s, m, h, t, u, n, i, e, b, r

inorder traversal: p, j, q, f, c, k, g, a, d, r, b, h, s, m, e, i, t, n, u


Example
r
a
b
c
d
e
f
g
h
i
j
k
m
n
p
q
s
t
u
preorder traversal: r, a, c, f, j, p, q, g, k, d, b
e, h, m, s, i, n, t, u
postorder traversal: p, q, j, f, k, g, c, d, a, s,
m, h, t, u, n, i, e, b, r
inorder traversal: p, j, q, f, c, k, g, a, d, r, b
h, s, m, e, i, t, n, u
保持室內1.5公尺以上
社交距離或戴口罩
Please keep the social distance
of 1.5M indoors or wear a mask.
COVID-19
關鍵防疫時刻
共同守護校園健康