

Rooted Trees

Def A rooted tree is called a m-ary tree if every internal vertex has no more than m children.

The tree is called a complete m-ary tree if every internal vertex has exactly m children.

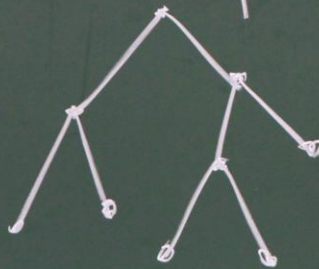
Remark

$m = 2$: binary

$m = 3$: ternary

$m = 4$: quaternary

Example a complete binary tree



Property Let T be a complete m -ary tree with n vertices, in which there are l leaves and i internal vertices. Then

1. $n = m\hat{i} + 1$

2. $l = (m-1)\hat{i} + 1$

3. $\hat{i} = \frac{l-1}{m-1} = \frac{n-1}{m}$

Proof 1. Every vertex, except the root, is a child of an internal vertex. Since each of \hat{i} internal vertices has m children, there are $m\hat{i}$ vertices in the tree other than the root. Therefore, the tree contains $n = m\hat{i} + 1$ vertices.

2. $l = n - \hat{i} = m\hat{i} + 1 - \hat{i} = (m-1)\hat{i} + 1$

3. From 1 and 2, $\hat{i} = \frac{l-1}{m-1} = \frac{n-1}{m}$. \square



Def A rooted tree of height h is called balanced if the level number of every leaf is $h-1$ or h .

Property For an m -ary tree of height h with l leaves, $l \leq m^h$ and $h \geq \lceil \log_m l \rceil$.

Remark $\lceil x \rceil$ is the smallest integer $\geq x$.
 $\lfloor x \rfloor$ is the largest integer $\leq x$.

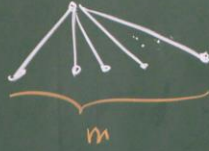


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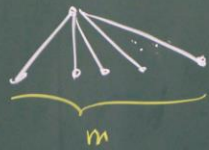
Remark $\lceil x \rceil$ is the smallest integer $\geq x$.
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Proof The proof is by induction on h .
For $h=1$, $l \leq m = m^h$.

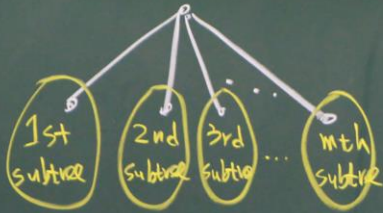


Now assume that the result is true for all m -ary trees of height $< h$.
Consider the subtrees at vertices of level 1.
There are at most m such subtrees.

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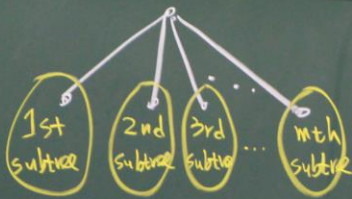


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Each subtree has height $\leq h-1$.
 So by the induction hypothesis, each subtree has $\leq m^{h-1}$ leaves.
 Hence there are at most $m \cdot m^{h-1}$
 $= m^h$ leaves in T . This completes the
 induction argument.

Since h is an integer and $l \leq m^h$, $h \geq \lceil \log_m l \rceil$. \square



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Property For a balanced complete m -ary tree with l leaves, $h = \lceil \log_m l \rceil$.

Proof Since each leaf is of level h or $h-1$ and the height is h , there is at least one leaf of level h . It follows that there are more than m^{h-1} leaves.

We then have $m^{h-1} < l \leq m^h$.

$$\Rightarrow h-1 < \log_m l \leq h.$$

Hence $h = \lceil \log_m l \rceil$. □

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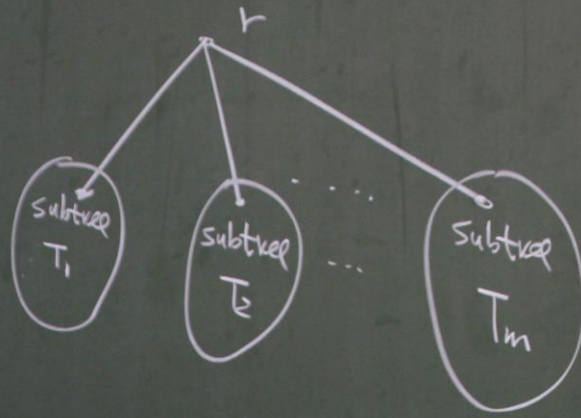
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Hence $h = \lceil \log_m l \rceil$. □

Tree Traversal

Consider a rooted tree T with root r .



Preorder traversal

1. Visit r
2. Traverse T_1 in preorder.
3. Traverse T_2 in preorder.
- ⋮
- mth. Traverse T_m in preorder.

Preorder traversal

1. Visit v
2. Traverse T_1 in preorder.
3. Traverse T_2 in preorder.
- ⋮
- mth. Traverse T_m in preorder.

Postorder traversal

1. Traverse T_1 in postorder.
2. Traverse T_2 in postorder.
- ⋮
- m. Traverse T_m in postorder
- mth. Visit v .

Postorder traversal

1. Traverse T_1 in postorder.
 2. Traverse T_2 in postorder.
 - ⋮
 - m. Traverse T_m in postorder
- mfl. Visit v .

Inorder Traversal

1. Traverse T_1 in inorder.
2. Visit v .
3. Traverse T_2 in inorder.
- ⋮
- mfl. Traverse T_m in inorder.

Inorder Traversal

order.

order.

order

1. Traverse T_1 in inorder.

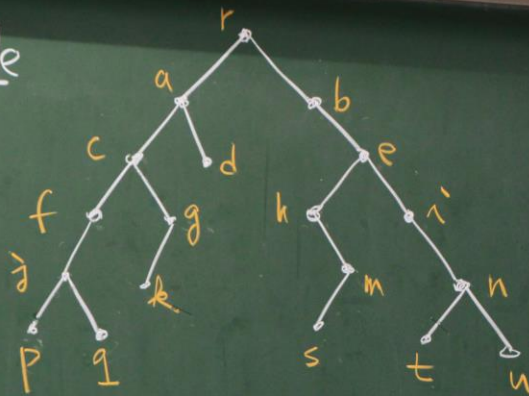
2. Visit r .

3. Traverse T_2 in inorder.

⋮

$m+1$. Traverse T_m in inorder.

Example



preorder traversal: $r, a, c, f, j, p, q, g, k, d, b$

e, h, m, s, i, n, t, u

postorder traversal: $p, q, j, f, k, g, c, d, a, s,$

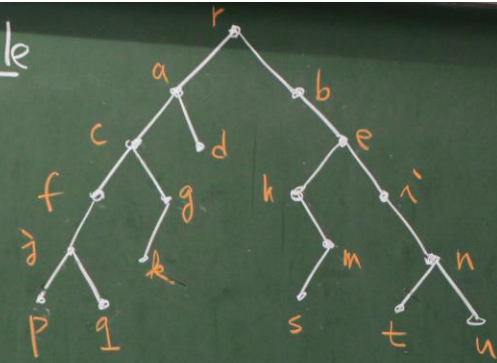
inorder traversal: $p, j, i, f, c, k, g, a, d, r, b$
 h, s, m, e, i, t, n, u

保持室內
社交距
Please keep th
of 1.5M indoor

關關防
共同守
戴口罩、上
出入室請
洗手五步

Example

er.
der.
der.



preorder traversal: r, a, c, f, j, p, q, g, k, d, b

postorder traversal: p, q, j, f, k, g, c, d, a, s, e, h, m, s, i, n, t, u

inorder traversal: p, j, q, f, c, k, g, a, d, r, b, h, s, m, e, i, t, n, u

保持室內1.5公尺以上
社交距離或戴口罩
Please keep the social distance
of 1.5M indoors or wear a mask

COVID-19
注意事項
1. 溫度: 38.5
2. 濕度: 37.5
請立即通報 醫
務中心
電話: 02-074300
傳真: 02-074300
02-074300

開關防疫時刻
共同守護校園健康
減少接觸口鼻
避免握手、上落樓梯、扶欄等
出入圖書館及公共場所前後，須執行噴霧
消毒雙手及消毒
洗手去字號